PRODUCTION INCREASE OF HEAVY OILS BY ELECTROMAGNETIC HEATING

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Production Increase of Heavy Oils By Electromagnetic Heating

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Abstract

Several methods of accelerating oil flow by applying heat from the wellbore to the reservoir have been successfully field tested. However, it appears that the possibility of heating by high-frequency radiation has not been extensively investigated, either theoretically or experimentally.

The concepts involved are discussed, and a mathematical model is developed in order to evaluate the temperature distributions and other physical effects resulting from the radiation of electromagnetic energy into an oil reservoir. Flow performance is compared to the unheated case on the basis of a steady-state temperature distribution, which is derived. The results of a pre-steady-state transient flow condition are also presented.

Calculations based on the mathematical model developed indicate that further work is warranted to prove or disprove the feasibility of this method of stimulation. A brief discussion of completion and engineering problems is included.

Introduction

FROM TIME TO TIME, the technical literature has made reference to heat stimulation by the radiation of electromagnetic energy from the wellbore. A Russian paper[1] touched on one aspect of the subject, and compared electromagnetic and contact heating for a shut-in well, using a spherically symmetric model. The possibilities of concurrent flow and power input were not discussed.

It should be emphasized that this paper deals not with conduction or induction, but true radiation, as exemplified by the output of a short-wave radio station. In the broadest terms, electromagnetic effects are exhibited from the lowest frequencies (e.g. 60 cycle and lower) to the very highest (X-rays, gamma radiation, etc.). For the practical purpose of wellbore heating, however, the following discussion should be viewed in terms of those frequencies at which high power can be generated on a continuous basis. At the present, this includes frequencies up to about 20000 megacycles per second. Figure 1 shows schematically the mode of operation envisioned for radiation heating.

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Theory

THE TEMPERATURE DISTRIBUTION

Power Absorption and the Heat Balance

For a linear homogeneous conducting medium, plane radiation propagating in the +x direction will be absorbed according to the following relationship:

$$\frac{d\Phi(x)}{dx} = -\alpha\,\Phi(x) \quad (1)$$

where $\Phi(x)$ = power density (watts/cm^2)
x = position coordinate (cm)
α = power absorption coefficient (1/cm).

The power absorption coefficient (α) depends on the properties of the absorbing medium in the following manner [2]:

$$\alpha = .02\,\alpha_e$$

and

$$\alpha_e^2 = \frac{\omega^2\mu\varepsilon}{2}\left\{\left(1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2\right)^{1/2}-1\right\} \quad (2)$$

where α_e = electric field absorption coefficient (l/meter)
σ = conductivity (mho/meter)
μ = permeability (H/meter)
ε = permittivity (F/meter)
ω = angular frequency ($2\pi \times$ frequency).

As is usual in the treatment of wellbore phenomena, radial symmetry will be assumed, and Equation 1 then becomes

$$\frac{d\Phi(r)}{dr} = -\left(\alpha + \frac{1}{r}\right)\Phi(r) \quad (3)$$

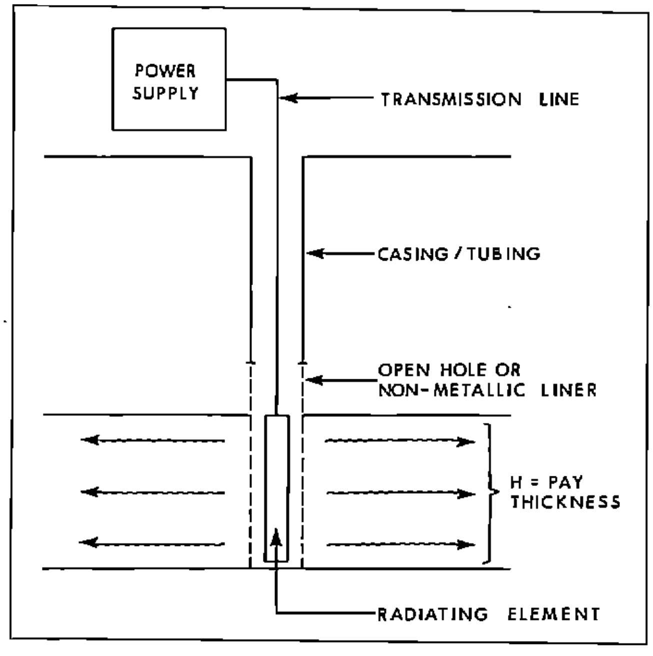

FIGURE 1 — Possible configuration of high-frequency heating system.

If we define $P(r)$ as the total power radiated across the radius r, then for a cylinder of height h,

$$P(r) = 2\pi r h \, \Phi(r) \quad (4)$$

Differentiating Equation 4 by r and using Equation 3 then gives

$$\frac{dP(r)}{dr} = -\alpha P(r) \quad (5)$$

which has the solution

$$P(r) = P_o e^{-\alpha(r-r_o)} \quad (6)$$

If r_o is the wellbore radius, then $P_o = P(r_o)$ is the total power radiated (watts). Therefore, the cylindrical element $(r, r+dr)$ will gain in heat content at the rate

$$\left(\frac{dQ}{dt}\right)_{radiation} = \frac{\alpha P(r)\, dr}{4.18} \text{ (cal/second)} \quad (7)$$

If it assumed that concurrent with the radiation absorption there is radial oil flow at the rate of dm_o/dt grams/second toward the wellbore, then the heat input by convection will be

$$\left(\frac{dQ}{dt}\right)_{convection} = - S_o \frac{d\,m_o}{dt} \left[T(r) - T(r + dr)\right] \quad (8)$$

where

S_o = specific heat of oil (cal/gm/°C)

The term $T(r)$ in Equation 8 is due to oil moving out of the element $(r, r+dr)$ at the temperature $T(r)$, and the term $T(r+dr)$ is due to oil moving into this elemental volume at the temperature $T(r+dr)$. From the definition of the derivative, we note that

$$T(r) - T(r + dr) = - \left(\frac{\partial T}{\partial r}\right) dr \quad (9)$$

Because: $\dfrac{dm_o}{dt} = \rho_o q_o$

where

ρ_o = oil density (gm/cc)
q_o = flow rate (cc/sec)

then Equation 8 can be rewritten

$$\left(\frac{dQ}{dt}\right)_{convection} = \rho_o q_o S_o \left(\frac{\partial T}{\partial r}\right) dr \quad (10)$$

Finally, the radial temperature distribution, assuming cylindrical symmetry, will result in a conduction term

$$\left(\frac{dQ}{dt}\right)_{conduction} = 2\pi h K \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) dr \quad (11)$$

where K = total heat conductivity (cal/sec/°C/cm)

The sum of the radiation, convection and conduction terms given by Equations 7, 10 and 11 therefore gives the net heat input rate into the cylindrical element $(r, r + dr)$:

$$\left(\frac{dQ}{dt}\right)_{net} = \left\{\frac{\alpha P(r)}{4.18} + \rho_o q_o S_o \left(\frac{\partial T}{\partial r}\right) + 2\pi h K \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)\right\} dr \quad (12)$$

The net heat input given by Equation 12 therefore determines the temperature change in the cylindrical element $(r, r + dr)$ which has the volume $2\pi\, r h dr$.

Define:

$$\rho_t = \rho_r (1 - \phi) + \rho_o \phi (1 - \sigma_w) + \rho_w \phi \sigma_w \quad (13)$$
$$S_t = (\rho_r S_r (1 - \phi) + \rho_o S_o \phi (1 - \sigma_w) + \rho_w S_w \phi \sigma_w)/\rho_t$$

where

ϕ = rock porosity
σ_w = connate water saturation
ρ = density (gm/cm³)
S = specific heat (cal/gm/°C)
r,o,w,t, = rock, oil, water, total

Then $\rho_t S_t$ is the specific heat per unit volume of reservoir rock and fluid.

Therefore

$$\left(\frac{dQ}{dt}\right)_{net} = 2\pi r h \rho_T S_T \left(\frac{\partial T}{\partial t}\right) dr \quad (14)$$

Equating the right-hand sides of Equation 12 and 14, dividing through by dr and neglecting the heat conduction term in Equation 12 then gives

$$2\pi r h \rho_t S_t \left(\frac{\partial T}{\partial t}\right) = \frac{\alpha P(r)}{4.18} + \rho_o q_o S_o \left(\frac{\partial T}{\partial r}\right) \quad (15)$$

where

$$\rho_t S_t = \rho_r S_r (1 - \phi) + \phi \rho_w S_w \sigma_w + \phi \rho_o S_o (1 - \sigma_w) \quad (16)$$

Finally, substituting for $P(r)$ in Equation 15 by using Equation 6 gives

$$\frac{\partial T}{\partial t} = \frac{1}{2\pi r h \rho_t S_t} \left\{\frac{\alpha P_o e^{-\alpha(r-r_o)}}{4.18} + \rho_o q_o S_o \left(\frac{\partial T}{\partial r}\right)\right\} \quad (17)$$

Equation 17 must now be solved for the temperature distribution $T(r,t)$ in order to ascertain the effect of radiative heating on the flow rate of the well.

(a) Steady-State Temperatures

When the left-hand side of Equation 17 equals zero, the resulting time-independent differential equation defines the steady-state system. If the resulting temperature distribution is denoted by $\theta(r)$, then

$$\frac{\partial \theta}{\partial r} = - \frac{\alpha P_o e^{-\alpha(r-r_o)}}{4.18 \rho_o q_o S_o} \quad (18)$$

which can be integrated immediately to give

$$\theta(r) = \theta(r_o) + \frac{P_o}{4.18 \rho_o q_o S_o} \left[e^{-\alpha(r-r_o)} - 1\right] \quad (19)$$

Because no heating occurs at $r = \infty$, then $\theta(\infty)$ must equal the initial reservoir temperature T_o, therefore

$$\theta(r_o) - T_o = \frac{P_o}{4.18 \rho_o q_o S_o} \quad (20)$$

Equation 20 expresses the steady-state temperature rise at the wellbore. It is simply an expression of the heat balance at the wellbore and is a necessary but not sufficient condition for defining a steady-state temperature distribution. Combining Equations 19 and 20 then gives the final temperature distribution

$$\theta(r) = T_o + \frac{P_o e^{-\alpha(r-r_o)}}{4.18 \rho_o q_o S_o} \quad (21)$$

Figure 2 shows this distribution in graphical form.

(b) Transient Temperatures — Constant Flow

This is the case for which the temperature is steadily increasing, with the flow rate held constant at q_o. Rearranging Equation 18 and substituting it for the first term on the right-hand side of Equation 17 gives

$$\left(\frac{\partial T}{\partial t}\right) = \frac{\rho_o q_o S_o}{2\pi h \rho_t S_t r}\left\{\frac{\partial T}{\partial r} - \frac{\partial \theta}{\partial r}\right\} \quad \ldots \ldots \quad (22)$$

Making the substitutions $U = \theta - T$ and

$A = \dfrac{\rho_o q_o S_o}{2\pi h \rho_t S_t}$, and noting that $\partial\theta/\partial t = 0$, then

$$\frac{\partial u}{\partial t} = \frac{A}{r}\frac{\partial u}{\partial r} \quad \ldots \quad (23)$$

Equation 23 has the formal solution

$$U(r, t) = f(r^2 + 2At) \quad \ldots \quad (24)$$

The form of the function remains unknown. However, at $t = 0$, $T(r,0) = T_o$, and we obtain the first initial condition

$$U(r, o) = \frac{P_o e^{\alpha r_o} e^{-\alpha r}}{4.18\, \rho_o q_o S_o} \quad \ldots \quad (25)$$

The other initial condition is that $(\partial T/\partial r) = 0$ at $t = 0$. Therefore, putting the first term in Equation 22 equal to zero and using Equation 18 gives the second initial condition

$$\frac{\partial U(r, o)}{\partial t} = \frac{-\alpha P_o e^{-\alpha(r - r_o)}}{4.18\,(2\pi h \rho_t S_t) r} \quad \ldots \quad (26)$$

At $t = 0$, Equation 24 gives

$$U(r, 0) = f(r^2) \quad \ldots \quad (27)$$

Comparing Equation 27 with Equation 25 indicates that $f(r^2)$ has the form

$$f(r^2) = k\, e^{-\alpha\sqrt{r^2}} \quad \ldots \quad (28)$$

Therefore, we can conclude that

$$U(r, t) = f(r^2 + 2At) = k\, e^{-\alpha\sqrt{r^2 + 2At}} \quad \ldots \quad (29)$$

It can be shown that Equation 29 also satisfies the second initial condition given by Equation 26, as well as the obvious conditions

$$\frac{\partial U(r, \infty)}{\partial t} = U(r, \infty) = 0 \quad \ldots \quad (30)$$

Hence, the transient temperature distribution, in the case of a steady flow rate to the wellbore, is given by

$$T(r, t) = T_o + \frac{P_o e^{\alpha r_o}}{4.18\, \rho_o q_o S_o}\left\{e^{-\alpha r} - e^{-\alpha\sqrt{r^2 + 2At}}\right\} \quad (31)$$

Figure 3 shows the transient temperature distribution after 2.2 days for different steady flow rates.

(c) Transient Temperatures — No Flow

Taking the limit of Equation 31 as the flow rate goes to zero (note that A depends on q) gives the result

$$T(r,t) = T_o \frac{\alpha P_o e^{-\alpha(r - r_o)} t}{4.18\,(2\pi h \rho_t S_t)\, r} \quad \ldots \quad (32)$$

This distribution, which is the same as is obtained by integrating Equation 17 with $q = 0$, has no steady state.

(d) Transient Temperatures — Increasing Flow

This case cannot be handled analytically for an arbitrary time-dependent flow rate $q_o(t)$. Equation 17 must be integrated numerically using finite time steps in conjunction with the flow rate calculated or assumed for the corresponding period of time. For a realistic calculation, the flow rate would be constrained by the pressure drawdown and the temperature/viscosity distribution during the time increment.

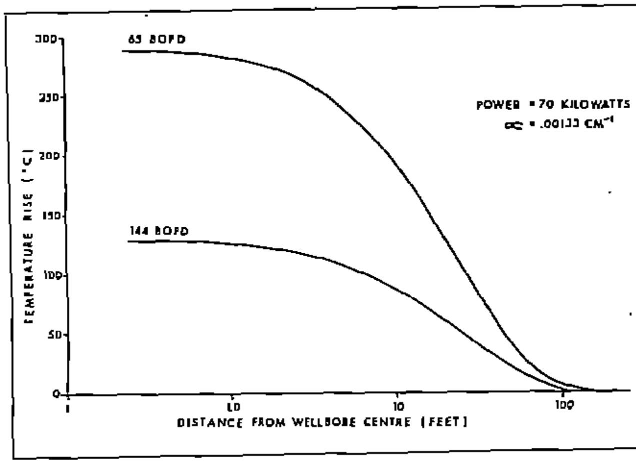

FIGURE 2 — Steady-state temperature distribution.

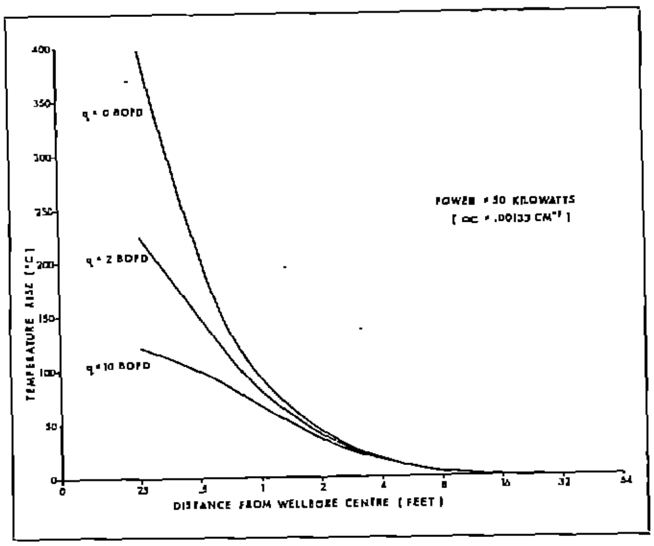

FIGURE 3 — Temperature rise after 2.2 days.

Flow Rate Calculations

The differential equation relating the temperature gradient, rate of temperature change and flow rate, given by Equation 17, is essentially no more than a heat balance. As such, it is a necessary but not sufficient condition for the calculation of the flow rate itself. Another relationship between the temperature and rate of flow is needed, and this is provided by Darcy's law in conjunction with a detailed knowledge of the viscosity-temperature relationship for the reservoir fluid.

Darcy's law for a radial system is given by

$$q(r) = \frac{-2\pi r h k}{\mu(r)}\frac{dp}{dr} \quad \ldots \quad (33)$$

where
$q(r)$ = flow rate at r (cc/sec.)
r = radius (cm)
h = thickness of flowing zone (cm)
k = Permeability (darcies)
$\mu(r)$ = viscosity at r (centipoise)
$\dfrac{dp}{dr}$ = pressure gradient (atmospheres/cm)

Taking r_e as the radius of the external boundary of the flow and q at the flow rate at the wellbore, and assuming the relationship for a compressible fluid[3],

$$q(r) = q(1 - r^2/r_e^2),$$

then Equation 33 can be integrated to give

$$Pe - P_o = -\frac{q}{2\pi hk}\int_{r_o}^{r_e}\frac{\mu(r)\,(1 - r^2/r_e^2)\,dr}{r} \qquad (34)$$

where r_o = wellbore radius
P_e = pressure at external radius
P_o = pressure at wellbore

In the case of no heating, $\mu(r) = \mu_o$ and

$$P_e - P_o = \frac{-\mu_o q}{2\pi hk}\,[\ln(r_e/r_o) - .5] \qquad (35)$$

assuming $r_e \gg r_o$

For the heated case, the integral in Equation 34 can be written as the sum of two integrals, and the equation becomes

$$P_e - P_o = -\frac{q'}{2\pi hk}(I_1 - I_2) \qquad (36)$$

where $I_1 = \int_{r_o}^{r_e}\frac{w(r)\,dr}{r}$

$I_2 = \int_{r_o}^{r_e}\frac{w(r)\,r\,dr}{r_e^2}$

q' = flow rate after heating

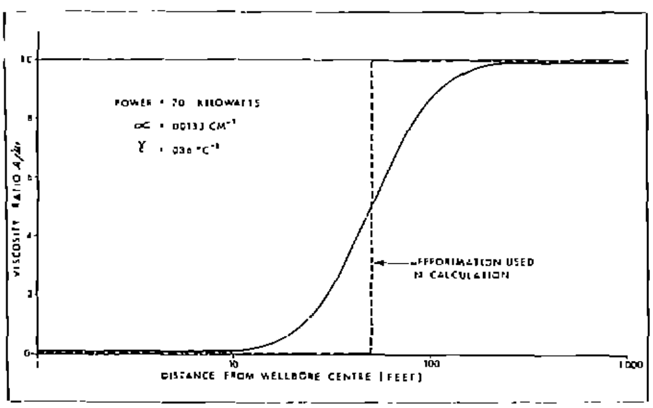

FIGURE 4 — Viscosity ratio for steady-state temperature distribution.

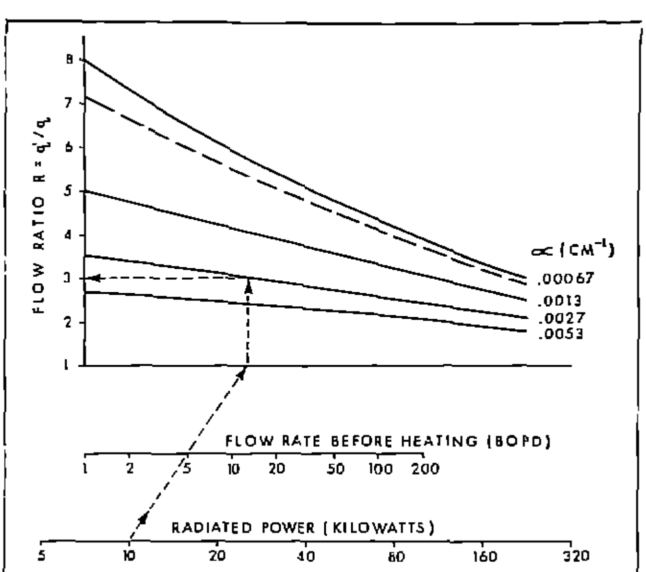

FIGURE 5 — Steady-state flow ratio.

Assuming that the pressure drawdown ($P_e - P_o$) is held constant, then dividing Equation 35 by Equation 36 gives the flow ratio $R = q'/q$ as

$$R = \frac{\mu_o\,[\ln(r_e/r_o) - .5]}{I_1 - I_2} \qquad (37)$$

To evaluate the integrals I_1 and I_2 we must assume that the viscosity of the oil is known as a function of the temperature, and that the temperaure distribution is itself known.

As an example, we will use a viscosity-temperature relationship of the form

$$\mu(T) = \mu_o\,e^{-\gamma(T - T_o)} \qquad (38)$$

where the subscript O refers to initial reservoir conditions.

If we assume a steady-state temperature distribution as given by Equation 21, the resulting integrals are not amenable to evaluation in any simple closed form. However, very good approximations can be made.

Figure 4 shows typical viscosity distributions based on the steady-state temperature profiles. If we define r_c to be the radius at which the value of the viscosity falls to half its initial value w_o, then visual inspection of Figure 4 shows the viscosity to be very nearly skew-symmetric about the value r_c, when r is on a logarithmic scale. Therefore, a minimum of error should be introduced if the integral I_1 is approximated by using

$$\mu(r) = 0,\ r_o < r < r_c$$

$$\mu(r) = \mu_o,\ r_c < r < r_e$$

Then

$$I_1 = \mu_o \ln(r_e/r_c) \qquad (39)$$

Using the same approximation, the second integral becomes

$$I_2 = \mu_o/2\,(1 - r_c^2/r_e^2) \qquad (40)$$

For the case of $r_c \ll r_e$, which will always be true in practical applications, the bracketted terms in Equation 40 will reduce to unity, therefore Equation 37 becomes

$$\frac{q'}{q} = R = \frac{\ln(r_e/r_o) - .5}{\ln(r_e/r_c) - .5} \qquad (41)$$

The radius r_c, although defined rather arbitrarily in terms of the viscosity distribution, thus has the mathematical interpretation of being the effective wellbore radius for the case in which the steady-state temperature distribution is reached.

(a) Steady-State Flow Rates

The use of Equation 41 requires a value of r_c. If Equation 38 is evaluated for $\mu = \mu_o/2$ and the resulting temperature (T) is substituted into Equation 21, the result is a relationship between r_c and the other physical parameters:

$$r_c = \frac{1}{\alpha}\ln\left\{\frac{P_o}{5.32\,\rho_o\,q'\,S_o}\right\} \qquad (42)$$

assuming $r_c \gg r_o$, and q' in BOPD

An estimate for q′ in Equation 42 results in a value for r_c. This is then substituted into Equation 41. The results for a reasonable range of initial flow rates, input power and absorption coefficients are shown in Figure 5. Values of .25 and 660 feet were used for r_o and r_e respectively, and $\gamma = .072/°C$; $\rho_o S_o = 0.5$ cal/

cc/°C. The dashed line on Figure 5 is for $\alpha = .00079$ as given in the Russian paper[1].

(b) Transient Flow Rates

The calculation of flow rates during the transient (pre-steady-state) period is more difficult than calculations for the steady-state case. Equation 17 must be rewritten in incremental form, and short time increments used, in order to account for the changing flow rate (the analytic solutions obtained above assumed steady flow rates). The temperature distribution can then be found at the end of the time interval for the assumed flow rate. The resulting viscosity distribution is then used to evaluate Equation 37 numerically, and a new value of q′ is obtained. This value is then used again in the incremental equation for the same time step, and the process is repeated until convergence on q′ is obtained for that time step.

In this way, a flow rate is obtained for each time step. The integral I_1 in Equation 37 cannot be approximated, as in the steady-state case, because for the early stages of heating $r_e \cong r_o$. Figure 6 shows typical flow rate buildups. The parameters used were:

γ = .072/°C
$\rho_o S_o$ = .5 cal/cc/°C
$\rho_t S_t$ = .66 cal/cc/°C
h = 600 cm

(c) Comparison with Contact Electric Heating

Raising the temperature of the wellbore area by means of bottom-hole (contact) heaters is a well-established technique used for dewaxing and viscosity control in the tubing string. As a stimulation technique, this heating method suffers from the fact that the heat input is entirely due to conduction against the flow of oil to the wellbore. The derivation of the steady-state temperature distribution for contact heating is given in Appendix I. Figure 7 compares the temperature distributions due to radiative and contact heating resulting from an initial rate of 4.3 BOPD and effective input power of 10 kw. Figure 8 shows the results for an initial rate of 13.3 BOPD and input power of 20 kw. Other parameters used were

γ = .072/°C
K = .002 cal/sed./°C/cm
α = .00266/cm
h = 600 cm
$\rho_o S_o$ = 0.5 cal/cc/°C

(d) Direct Electric Heating

This method of heating near the wellbore relies on the generation of heat by the I^2R loss of 60-cycle alternating current directly applied to the wellbore. Rate increases of up to 300% have been recorded under certain circumstances[4]. Power requirements and temperature profiles have not been calculated for comparison with radiative heating.

Discussion

THE MATHEMATICAL MODEL

As in all analytic models, certain simplifications have been made in order to render tractable results. The most important of these assumptions are:
(1) radial symmetry of radiation;
(2) no heat losses to the adjacent formations;
(3) strictly radial pressure distribution.

In general, antennae of a simple dipole type have a power output intermediate between radial and spherical symmetry; specifically designed arrays can result in angular distributions which closely approximate a

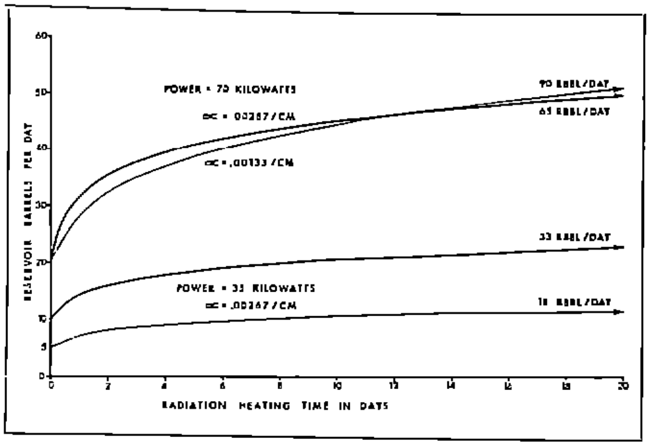

FIGURE 6 — Flow rate buildup vs heating time.

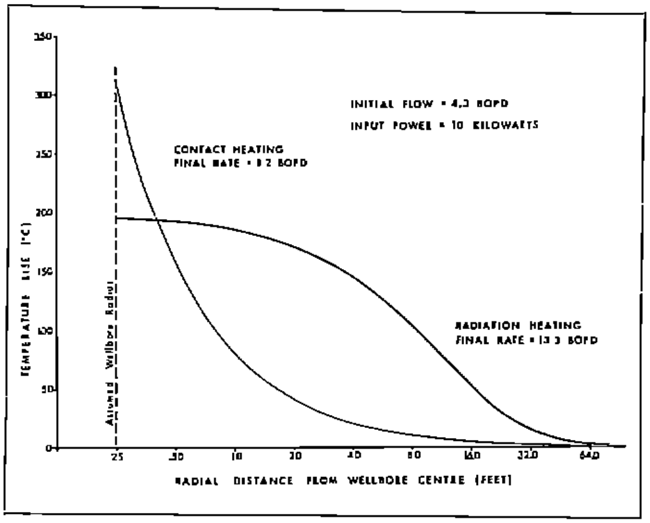

FIGURE 7 — Comparison of steady-state temperatures.

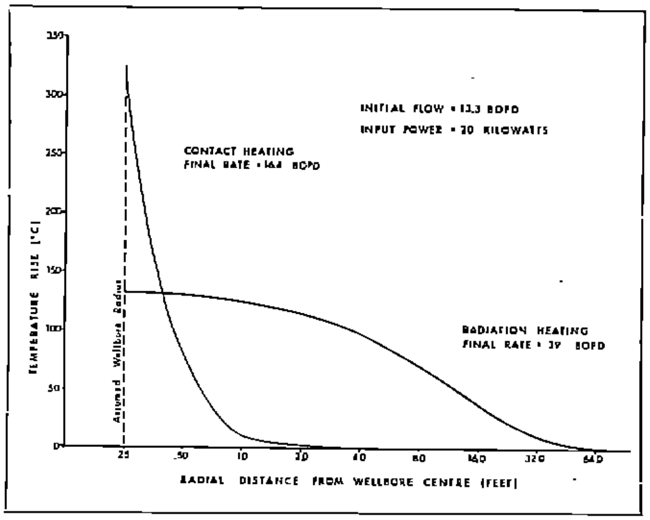

FIGURE 8 — Comparison of steady-state temperatures.

radial distribution. It is unlikely that this would be possible in the confined space of a wellbore. For practical purposes, it would be sufficient for the power to be angularly distributed in such a manner that it would be absorbed in the productive zone, rather than escaping to adjacent formations.

Quite apart from the problem of radiation leaving the productive formation is the problem of vertical heat conduction from the heated region. No attempt has been made to account for this loss of heat. It would appear, however, that in a practical operating situation an antenna of smaller vertical dimension than the thickness of the formation, placed midway in the formation, and radiating in a reasonably radial fashion, would give rise to a much smaller percentage of heat loss than, for instance, a steam stimulation.

PHYSICAL PARAMETERS

The parameter most in doubt, and also the most critical parameter in determining the feasibility of the method of electromagnetic heating, is the absorption coefficient α. A Russian paper[1] gives a value of the electric field absorption coefficient $\alpha_e = .0396$/meter. In terms of the absorption coefficient used in this paper, this corresponds to a value of .000792/cm, which appears encouraging. However, it is not known what frequency this value is for, or the nature of the reservoir rock for which the coefficient was measured.

POWER SUPPLY

Assuming that the reservoir parameters are such that the formation is susceptible to radiation stimulation, there remains the problem of supplying electromagnetic power to the formation.

At the present state of the art, power output is available in the range of 100 kilowatts to several megawatts throughout the frequency spectrum to about 20000 megacycles. Typical 60-cycle to r.f. conversion efficiency is of the order of 50% at the oscillator output. The size of such installations and auxiliary equipment such as cooling facilities would dictate their use at the wellhead.

With a high-frequency power source at the wellhead, a transmission line would be necessary to transmit the power to the formation. For frequencies up to the UHF range, a heavy coaxial cable would probably serve adequately, with fairly low power losses. Still higher frequencies require waveguides for efficient transmission, and the range of frequencies would be severely limited by the diameter of the tubing. It is expected, however, that these radar frequencies would have too high an absorption coefficient to be of use in oilfield heating applications, regardless of their proven use in microwave ovens.

The most serious engineering problems would occur at the transmitting element opposite the formation. Generally speaking, the size of an efficient radiating antenna increases with the wavelength used. Bearing in mind the typical dimensions of a wellbore and the pay zone thickness, low-frequency radiation efficiency could be very poor, regardless of the fact that such a frequency might be necessary to obtain a reasonably low power absorption coefficient. In addition to very severe restrictions on the antenna geometry, it must be remembered that high-frequency radiation cannot penetrate steel, therefore the bottom of the casing string would have to have a joint of plastic (or other dielectric) pipe, or the well would require an open-hole completion. High-temperature equipment would be required in either case, both for the flow string and the radiation hardware. The entire antenna system would have to be totally enclosed in some dielectric material, in order to prevent short-circuiting, which would occur if even the slightest amount of water were produced with the oil. Finally, the output impedance of the antenna and surroundings would have to be matched to the oscillator and transmission line in order to optimize power transfer from the power supply to the reservoir.

REVIEW OF RESULTS

It appears from Figure 6 that a near-steady-state temperature distribution would take many months to achieve if the flow rate was allowed to increase spontaneously. If the flow rate was restricted, however, Equation 31 can be used to calculate the temperature distribution, and near-steady-state conditions can be achieved faster, as more of the heat is retained in the formation.

Regardless of the approach to steady-state flow, it appears that under a wide variety of operating circumstances the flow rate can be expected to double within four weeks of the startup of radiation heating. This is important in that only a very limited field trial would be required to determine the feasibility of this stimulation method for a given oil reservoir.

It should be noted that all flow rates have been calculated as a flow ratio multiplied by the initial flow rate, due to the radial flow geometry used. The results on Figure 5 are therefore clearly not applicable to a case where there is no initial flow.

A further observation, based on the flow ratios from Figure 5, is that appreciable flow rate increases would be experienced even by wells initially producing at 20-100 BOPD rates. At the relatively modest power input of 20 kilowatts, 50% to 300% rate increases may be possible, depending on the power absorption coefficient α.

Conclusions

From a theoretical standpoint, it appears that heat stimulation of oil wells by radiation heating is an attractive possibility, for both the flow rate and the temperature distribution characteristics. It is likely that all hardware problems could be successfully accommodated, and therefore the technical feasibility of the method depends ultimately on the effective value of the power absorption coefficient. Measurements of power absorption as a function of frequency could be made in a laboratory, as could direct temperature measurements in a suitable medium surrounding a radiating antenna. The results of such experiments will be needed before further development of the method can be contemplated.

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APPENDIX I

The net heat transfer rate into a cylindrical element (r, r + dr) is given by Equation 12 above for concurrent radiation, convection and conduction:

$$\left(\frac{dQ}{dt}\right)_{net} = \left\{\frac{\alpha P(r)}{4.18} + \rho_o q_o S_o \left(\frac{\partial T}{\partial r}\right) + 2\pi hK \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)\right\} dr \quad \text{(A-1)}$$

The steady-state temperature distribution for the case of contact heating from the wellbore can be obtained by solving Equation A-1 for the conditions

$$\left(\frac{dQ}{dt}\right)_{net} = 0$$

and $P(r) = 0$ (i.e. no radiation)

Then from A-1

$$\left(\frac{\rho_o q_o S_o}{2\pi h K}\right) \frac{dT}{dr} = - \frac{d}{dr}\left(r \frac{dT}{dr}\right) \qquad \text{(A-2)}$$

Making the substitutions $A = \rho_o q_o S_o / 2\pi hK$

and

$$\psi = \frac{dT}{dr}$$

we obtain

$$A\psi = -\psi - r\frac{d\psi}{dr} \qquad \text{(A-3)}$$

Rearranging Equation A-3, we can obtain

$$\frac{d\psi}{\psi} = -(A + 1) \quad \frac{dr}{r} \qquad \text{(A-4)}$$

which can be integrated immediately to yield

$$(-A - 1)\ln(r) = \ln(\psi) + \text{constant}$$

or

$$\psi = \frac{dT}{dr} = Br^{-A-1} \qquad \text{(A-5)}$$

$$(\ln = \log_e)$$

Integrating Equation A-5 gives

$$T = -\frac{B}{A} r^{-A} \qquad \text{(A-6)}$$

or

$$T(r) = \frac{2\pi hK}{\rho_o q_o S_o} B r^{\frac{-\rho_o q_o S_o}{2\pi hK}} \qquad \text{(A-7)}$$

B is a constant which remains to be determined. Equation 20 above gives the steady-state temperature rise at the wellbore, which is independent of the method by which heat is introduced into the reservoir. In terms of $T(r)$, the temperature rise given by Equation A-7, Equation 20 becomes

$$T(r_0) = \frac{P_o}{4.18\, \rho_o q_o S_o} \qquad \text{(A-8)}$$

The constant B in Equation A-7 can therefore be evaluated and the steady-state temperature distribution is then known.